Mécanique des fluides : Étude qualitative et comportement asympotique des solutions

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Book of Abstracts

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Raphaël Danchin (Université Paris-Est Créteil)
Evelyne Miot (CNRS, Université Grenoble Alpes)

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Participants:

Didier Bresch (CNRS, Université de Savoie), *Didier.Bresch@univ-savoie.fr*
Frédéric Charve (Université Paris-Est Créteil), *frederic.charve@univ-paris12.fr*
Jean-Yves Chemin (Sorbonne université), *chemin@ann.jussieu.fr*
Dimitri Cobb (Université de Lyon 1), *dimitri.cobb@ens-rennes.fr*
Timothée Crin-Barat (Université Paris-Est Créteil), *timothee.crin-barat@u-pec.fr*
Raphaël Danchin (Université Paris-Est Créteil), *danchin@univ-paris12.fr*
Clément Denis (Université d'Aix-Marseille), *clement.denis@univ-amu.fr*
Anatole Gaudin (Université d'Aix-Marseille), *anatole.GAUDIN@univ-amu.fr*
Thierry Gallay (Université Grenoble Alpes), *Thierry.Gallay@univ-grenoble-alpes.fr*
Francesco Fanelli (Université de Lyon 1), *fanelli@math.univ-lyon1.fr*
Ludovic Godard-Cadillac (Sorbonne université), *ludovic.godard-cadillac@sorbonne-universite.fr*
Boris Haspot (Université Paris-Dauphine), *haspot@ceremade.dauphine.fr*
Jiao He (Université d'Evry), *jiao.he@univ-evry.fr*
Evelyne Miot (CNRS, Université Grenoble Alpes), *evelyne.miot@univ-grenoble-alpes.fr*
Sylvie Monniaux (Université d'Aix-Marseille), *sylvie.monniaux@univ-cezanne.fr*
Ivan Moyano (Université de Nice), *Ivan.Moyano@unice.fr*
Piotr B. Mucha (University of Warsaw), *mucha@hydra.mimuw.edu.pl*
Charlotte Perrin (CNRS, Université d'Aix-Marseille), *charlotte.perrin@univ-amu.fr*
Frédéric Rousset (Université Paris Saclay), *frederic.rousset@universite-paris-saclay.fr*
Didier Smets (Sorbonne université), *smets@ann.jussieu.fr*
Franck Sueur (Université Paris VI), *franck.sueur@math.u-bordeaux.fr*
Changzhen Sun (Université Paris Saclay), *changzhen.sun@universite-paris-saclay.fr*
Jin Tan (Université Paris-Est Créteil), *jin.tan@u-pec.fr*
Marc-Antoine Vassenet (Université Paris-Dauphine), *marc-antoine.vassenet@psl.eu*
Klaus Widmayer (Ecole Polytechnique Fédérale de Lausanne), *klaus.widmayer@epfl.ch*
Shan Wang (Université Paris-Est Créteil), *shan.wang@u-pec.fr*

TALKS

Mathematical modeling of compressible fluid mixtures by homogenization

Didier Bresch

CNRS, Université de Savoie
DIDIER.BRESCH@UNIV-SAVOIE.FR

During this talk, I will discuss the derivation of compressible fluid mixture models. A particular solution framework must be considered and continuous or discrete approaches may be proposed. This will be mainly based on joint works with C. Burtea, M. Hillairet and F. Lagoutière. We hope this will present an introduction on the subject for beginners.

Ekman boundary layers with large topography

Jean-Yves Chemin
Sorbonne université
CHEMIN@ANN.JUSSIEU.FR

In this talk, we investigate the asymptotic of the fast rotating Coriolis-Navier-Stokes system in the case of topography of size one. In this situation, the classical Taylor Proudman theorem leads to make the hypothesis of cylindrical symmetry for the topography and for the flow. Under this hypothesis, we establish a convergence result where the pumping Ekman term depends on the topography through the depth and the curvature of the ground.

Statistical solutions to the barotropic Navier-Stokes equations

Francesco Fanelli

Université de Lyon 1
FANELLI@MATH.UNIV-LYON1.FR

In this talk we are concerned with the notion of *statistical solutions* to models from fluid mechanics. We consider the barotropic Navier-Stokes equations, supplemented with non-homogeneous boundary data.

Statistical solutions are, roughly speaking, ensemble averages of trajectories emanating from a given distribution of initial data. In the first part of the talk, we study their dynamical properties, in particular their existence and stability. In the second part of the talk, we focus on the special class of *stationary* statistical solutions, and explore their role in the investigation of the validity of the so-called *ergodic hypothesis*.

This talk is based on joint works with *Eduard Feireisl* (Czech Academy of Sciences) and *Martina Hofmanová* (Universität Bielefeld).

Arnold's variational principle and its application to the stability of viscous planar vortices

Thierry Gallay

Université Grenoble Alpes
THIERRY.GALLAY@UNIV-GRENOBLE-ALPES.FR

We revisit the variational approach to nonlinear stability of planar flows, which was developed by V. I. Arnold around 1965. In particular, we study the coercivity properties of the quadratic form that describes the second variation of the energy at a radially symmetric vortex with strictly decreasing vorticity profile. We also show that this quadratic form can be used to obtain a new proof of nonlinear stability for the Lamb-Oseen vortices, which are self-similar solutions of the two-dimensional Navier-Stokes equations. This is all joint work with V. Sverak.

Collapses of point-vortices for Euler and quasi-geostrophic models

Ludovic Godard-Cadillac

Sorbonne université
LUDOVIC.GODARDCADILLAC@UNITO.IT

The point-vortex model is a standard model for planar incompressible inviscid fluid mechanics. It gives account of the situation where the vorticity is sharply concentrated around a finite number of points and then can be approximated by Dirac masses evolving in time. This model leads to a system of ordinary differential equations for N points evolving on the plane. This evolution system is well-defined as long as the point-vortices remain at positive distance from each other. Otherwise, the system is said to show a collapse trajectory and the velocity of the point-vortices blows-up near the time at which this collapse occurs.

In this talk, I will present recent studies on the collapses of vortices and more particularly on the trajectories of the vortices leading to a collapse. This involves boundedness results, convergence results, Hölder estimates and results about the improbability of collapses.

Wave-structure interactions, oscillating water columns in shallow water

Jiao He

Université d'Evry
JIAO.HE@UNIV-EVRY.FR

Wave energy converters (WECs) are devices that convert the energy associated with a moving ocean wave into electrical energy. In this talk we present a mathematical model of a particular wave energy converter, the so-called oscillating water columns in shallow water regime. This model can be reformulated as two transmission problems: one is related to the wave motion over the stepped topography and the other one is related to the wave-structure interaction where a fixed partially immersed structure is installed. We analyze the evolution of the contact line between the surface of the water and the surface of the structure and we show how to formulate the transmission problem as an initial boundary value problem with a semilinear boundary condition given by an ordinary differential equation. A rigorous well-posedness result of this coupled PDE-ODE will be given as well. Finally, we use the characteristic equations of Riemann invariants to obtain the discretized transmission conditions and show some numerical simulations in the end.

Burgers equation on graphs

Piotr Mucha

University of Warsaw
MUCHA@HYDRA.MIMUW.EDU.PL

Consider the classical inviscid Burgers equation

$$u_t + uu_x = 0$$

in the monodimensional case. The basic interpretation of the system explains the motion of one wave, creation of shocks and rarefaction waves. However the theory does not capture interaction of waves. Big waves eat smaller one. We cannot obtain a passing through phenomena between waves.

Our idea is to extend the monodimensional structure of the domain to a graph, giving possibility for the solution to take different paths. This model is a motivation to introduce a theory of the Burgers equation on metric graphs. The crucial point is the behavior of solutions at vertices, which must be suitably determined. I will show some interesting examples as well as key points of the mathematical theory.

The talk is based on joint results with Aleksandra Puchalska (Warszawa).

Time decay rates for kinetic equations and waves undergoing localised collisions and rough damping

Ivan Moyano

Université de Nice côte d'Azur
IVAN.MOYANO@UNICE.FR

In this talk we revisit the connection between the time decay rates for the damped wave equation and the geometric structure of the damping region originally studied by Lebeau in 1996: if the damping region satisfies the geometric control condition (Bardos, Lebeau and Rauch 1992), then we can expect an exponential decay of the energy. This result is sharp as if this geometric condition is not satisfied, the decay must be slower. In the worst case, when the region is a measurable set with positive measure in a compact Riemannian manifold, we prove (Burq and M., 2021) that the decay is at least logarithmic. This geometric condition condition, naturally rephrased in the phase space, has been exploited in the context of kinetic equations as the linear Boltzmann equation in the spirit of Villani's hypocoercivity in a localised collision regime. Some authors (Bernard and Salvarani 2016 and later Han-Kwan and Léautaud 2018) have shown exponential decay for the linear Boltzmann equation whenever the collision region satisfies the geometric control condition (and in fact, the control condition is also necessary for exponential decay) using non explicit methods. We give (Evans and M.,2019) a quantitative estimate of this exponential decay using some tools from Markov chains as the Döblin theorem.

Traveling waves for the porous medium equation in the incompressible limit

Charlotte Perrin

CNRS, Université d'Aix-Marseille
CHARLOTTE.PERRIN@UNIV-AMU.FR

In this talk, I will introduce a one-dimensional porous medium equation modeling the growth of living tissues. I will analyze the behavior and the stability of traveling waves solutions to this model in the so-called "incompressible limit". In this asymptotics, the pressure, which governs the diffusion process and limits the creation of cells in the tissue, becomes very stiff and the original PDE degenerates towards a free boundary problem of Hele-Shaw type. This is a joint work with Anne-Laure Dalibard and Gabriela Lopez-Ruiz.

Incompressible limit for free surface fluids

Frédéric Rousset

Université Paris-Saclay
FREDERIC.ROUSSET@UNIVERSITE-PARIS-SACLAY.FR

The aim of this series of talks will be to present a recent result with Changzhen Sun and Nader Masmoudi about the incompressible limit for the free surface Navier-Stokes system. A significant difficulty to address in the case of viscous fluids (even with fixed viscosity) in domains with boundaries is the presence of a boundary layer in the acoustic part. Before dealing with free surface fluids, we will thus recall some more classical results in domains with fixed boundaries (or even no boundaries) in order to explain the various phenomena in the system and the way they interact.

Stationary solutions to the incompressible Euler equations

Didier Smets

Sorbonne université
SMETS@ANN.JUSSIEU.FR

After recalling Arnold's (formal) interpretation of stationary solutions to Euler equations as critical points of the kinetic energy over leafs of isovortical vector fields, the course will go over three types of questions that have been tackled at least in some way based on this interpretation :

– ridigity and Liouville type theorems,

– existence and variational construction,

– tentative classification.

The focus will be restricted for the most part to regular solutions, letting aside important recent developments on "wild" solutions (in particular the course will not discuss convex integration). Questions related to stability will also be let aside because this is the object of Thierry Gallay's talk on Thursday.

2D incompressible Euler system in presence of sources and sinks

Franck Sueur

Université de Bordeaux
FRANCK.SUEUR@MATH.U-BORDEAUX.FR

This talk is devoted to the 2D incompressible Euler system in presence of sources and sinks. This model dates back to Viktor Yudovich in the sixties and is an interesting example of nonlinear open system which has been widely used in controllability theory within the scope of smooth solutions. In this talk we will review how the classical issues of existence and uniqueness of weak solutions are challenged by the presence of incoming and exiting vorticity.

Remarks on the stabilizing effect of anisotropic dispersive forces in the Euler equations

Klaus Widmayer

Ecole Polytechnique Fédérale de Lausanne
KLAUS.WIDMAYER@EPFL.CH

As is well documented in the corresponding literature, in geophysical fluids one naturally encounters dispersive forces. While dispersion is often easily identified in the associated linear models and for individual frequencies, it is a subtle effect overall and in particular leaves energy balances unaffected. In contrast to classical dispersive problems, in the geophysical setting one encounters anisotropies (and thus degeneracies) in the dispersive features themselves, e.g. through rotation or stratification. This further reduces available conservation laws and symmetries. Despite this, in certain settings one can show that even such anisotropic, relatively weak dispersive effects can exert a strong, stabilizing influence. Such results rely on a fine analysis of the nonlinear interactions.

In these talks I will give a –personally flavored and biased– introduction to some work in this direction, starting with the basic motivation of some of the geophysical models. The aim is to then give some samples of (what one might hope are) prototypical results, and to connect this with recent work and open problems.